Title	Additional Mathematics – Study Notes for Additional Mathematics Topic
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Title	Additional Mathematics (Quadratic Equation)
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Date	6/3/2018

Mentoring Club is once again bringing free notes to the public. Before I start, I want to give those "first-timer" some hints and basic overview on what is covered in this topic. Unlike Elementary Mathematics, Additional Mathematics quadratic equation requires the following skills to be mastered as objective, aside from the usual quadratic formula and factorization you all do.

1. Sum and Product Manipulation
2. Discriminant Manipulation

Formula and Inequalities needed to remember as part of this topic:

Sum of Roots

$$\alpha + \beta = \frac{-b}{a}$$

Product of Roots

$$\alpha\beta = \frac{c}{a}$$

Proof of Concept Given Below
Since Quadratic Equation have Two Roots, we substitute the two roots as algebraic values. $x = \alpha$ OR $x = \beta$ $x - \alpha = 0$ OR $x - \beta = 0$ $(x - \alpha)(x - \beta) = 0$ **When Expanded You Get** $\boldsymbol{x^2} - \alpha x - \beta x + \alpha\beta = 0$ And quadratic equations can be written in the following form $ax^2 + bx + c = 0$ (In this case $a = 1$)

We extract the following values to derive the sum of roots

$$-\alpha x - \beta x = \frac{b}{a}x$$

Divide both sides by x to get

$$-\alpha - \beta = \frac{b}{a}$$

Dividing both sides by -1 to get

$$-\mathbf{1}(-\alpha - \beta) = \frac{-b}{a}$$

Which can be simplified into

$$\alpha + \beta = \frac{-b}{a}$$

From the same equation, we can also derive the product of roots as well:

$$\boldsymbol{x}^{\mathbf{2}} - \alpha x - \beta x + \alpha\beta = 0$$

In this case, we see how $\boldsymbol{\alpha\beta}$ resembles $\boldsymbol{c}$ as

$$\boldsymbol{x}^{\mathbf{2}} - \alpha x - \beta x + \alpha\beta = ax^2 + bx + c = 0$$

Therefore $\alpha\beta = \frac{c}{a}$

Principle of Discriminant:

$D = b^2 - 4ac$

Condition	**Consequence and Interpretation**
$D > 0$	Real and Distinct Roots
$D = 0$	Real and Equal Roots
$D \geq 0$	Real Roots (In General)
$D < 0$	Unreal Roots (Or called No Real Roots)

(Questions all Taken from Online Sources)
[Sum and Product of Roots]
(Warning: Write your notation clearly and in a way your teacher can at first glance tell what you are writing)
Q1. The equation $2x^2 - x - 2 = 0$ has roots α and β. **Without calculating the actual values of** α and β.
Find the values of
(a) $(\alpha + 3)(\beta + 3)$
(b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$
(c) $(\alpha - \beta)^2$

Steps Taken	Reasoning (If any)
Q1[General] $\alpha + \beta = \frac{1}{2}$ $\alpha\beta = -\frac{2}{2} = -1$	[Needed for the (a), (b), and (c)]
(a) $\alpha + 3$ $\beta + 3$ Since $(\alpha + 3)(\beta + 3) = \alpha\beta + 3\alpha + 3\beta + 9$ Which can be factorized into the following expression. $\alpha\beta + 3(\alpha + \beta) + 9$ Sub in values obtained previously and you should get $(-1) + 3\left(\frac{1}{2}\right) + 9 = \frac{18}{2}$	

(b) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ Cross multiply the denominator to get $\frac{\alpha^2+\beta^2}{\alpha\beta}$ Which equates to the following values $\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$ Sub in previous values obtained and you should get $\frac{\left(\frac{1}{2}\right)^2-2(-1)}{(-1)}$ =2 $\frac{1}{4}$	
(c) $(\alpha-\beta)^2$ Expand to get the following $(\alpha^2-2\alpha\beta+\beta^2)$ Rearrange the expression into the following pattern $\alpha^2+\beta^2-2\alpha\beta$ Which can be factored into $\alpha^2+\beta^2-2(\alpha\beta)$ From Question 1(b) we already found out the value for $(\alpha^2+\beta^2) = -2\frac{1}{4}$ Alongside with value of αβ obtained earlier in the question. Therefore $\alpha^2+\beta^2-2\alpha\beta = -2\frac{1}{4}-2(-1) = -2\frac{1}{4}+2$ $= -\frac{1}{4}$	

Discriminant Manipulation

Question 2

Find the range of value for k for which the expression $3x^2 + 6x + k$ is always positive for all real value of x.

[When you see the following keywords: Always Positive, Always Negative, No Real Roots, it implies the graph does not have any contact with the X-axis, thus having no real roots and thus $b^2 - 4ac < 0$.]

Knowing such information, we will proceed on with finding a value of k that can satisfy the requirement of no real roots at all.

$6^2 - 4(3)(k) < 0$
$36 - 12k < 0$
$-12k < -36$
$12k > 36$
$k > 3$

Question 3

The expression $(k + 3)x^2 + 6x + k = 5$ has two distinct solutions for x.

(a) Show that k satisfy $k^2 - 2k - 24 < 0$
(b) Find the set of possible value of k

[When you see the following keywords, pass through graph at two distinct points, distinct roots or anything similar, it means the graph have two real and distinct roots, thus $b^2 - 4ac > 0$.]

(a)
$6^2 - 4(k + 3)(k - 5) > 0$

$36 - 4(k^2 + 3k - 5k - 15) > 0$
$36 - 4k^2 - 12k + 20k + 60 > 0$

$-4k^2 - 12k + 20k + 60 - 36 > 0$
$-k^2 - 3k + 5k + 24 > 0$

$-k^2 + 2k + 24 > 0$
$k^2 - 2k - 24 < 0$ [Shown]

(b)
$k^2 - 2k - 24 < 0$
Factorize left hand side to get
$(k - 6)(k + 4) < 0$

(Less than zero means k must be within a range of values)
Thus, we get
$k < 6$ OR $k < -4$

Which can be written as
$-4 < k < 6$

Question 3
Find the values of p for which the equation $3x^2 + 2px - p$ has
(a) Real and Equal Roots
(b) Distinct Roots

When you see the following keywords "has only 1 solution", "real and equal roots", "repeated roots" or anything similar the graph merely has contact with the X-axis one time, thus $b^2 - 4ac = 0$

(a)
$(2p)^2 - 4(3)(-p) = 0$
$4p^2 + 12p = 0$
$p(p + 3) = 0$
$p = -3$ or $p = 0$

(b)
$(2p)^2 - 4(3)(-p) > 0$
$4p^2 + 12p > 0$
(More than zero means, value must not be within a range which means)
$p > 0$ OR $p < -3$

Title	Additional Mathematics [‘N’ Levels] and Computing Mathematics Calculus - Basic Differentiation
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	5/4/2018

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- ‘O’ Level and ‘A’ Level (Trigonometric, Exponential and Logarithmic Function are not covered in this document)
- Engineering (Trigonometric Function Not Covered)
- Chemistry (Logarithmic and Exponential Function Not Covered)

However, if you haven’t studied additional mathematics before and want exposure to the topic before entering your course or before you enter Secondary 4, this should give you the basic concepts of how it works.

The following are covered in this document

Basic Overview of Differentiation

- Reason to study this topic
- Notation Guide

Differentiation Basic Rules

- Power Rule
- Sum Rule
- Difference Rule
- Product Rule
- Quotient Rule
- Chain Rule

Brief Overview of Differentiation (And Calculus in General)

Without going into too much detail about the history of calculus, calculus is a branch of Mathematics introduced to study about change. Differentiation is a sub-branch that deals with the gradient of a given function at every point on a curve.

Calculus is very important in the following field for the following reasons

- Computing – To study time complexity of programming algorithms and do various data analytics and estimation
- Physics – Majority of physical laws are derived with help of calculus (Newton used calculus to research how physics works and used that in his later works)
- Engineering – Engineering is closely related to physics as Engineering students study physics as their basic module before they carry on with intermediate modules
- Chemistry – Study speed of chemical reactions

And the list can go on and on

Notation	**Meaning**	**Explanation**	**Notes**
$F(x)$	Function of x		I also use $f(x), g(x)$ in certain situations
$F'(x)$	First Derivative of $f(x)$		I also use $f'(x)$, $g'(x)$ in certain situations
$F''(x)$	Second Derivative of $f(x)$	First derivative of $f'(x)$	I also use $f''(x), g''(x)$ in certain situations
$\frac{dy}{dx}$	First derivative of y with respect to x		
$\frac{d^2y}{dx^2}$	Second derivative of y with respect to x	First derivative of $\frac{dy}{dx}$	

Follow the question with regards to notation matters:

If the question mentions a function in the following form:

$$f(x) = ax^n + ax^{n-1} \dots + c$$

When answering the question, use $f'(x)$ for first derivative and $f''(x)$ for second derivative.

If the question mentions an equation in the following form:

$$y = ax^n + ax^{n-1} \dots + c$$

When answering the question use $\frac{dy}{dx}$ for first derivative and $\frac{d^2y}{dx^2}$ for second derivative

Power Rule
Given Function $f(x) = ax^n$
The derivative of the function is $f'(x) = anx^{n-1}$

Question 1.
Differentiate the following with respect to x

(a) $y = x^5$
(b) $y = \frac{1}{x^3}$
(c) $y = 3x^4$
(d) $y = \pi$

1(a)
$y = x^5$
Differentiate using power rule $\frac{dy}{dx} x^5 = 5x^4$

1(b)
$$y = \frac{1}{x^3}$$

Convert to index notation: $y = x^{-3}$
Differentiate using power rule: $\frac{dy}{dx} x^{-3} = -3x^{-4}$

1(c)
$y = 3x^4$
Differentiate using power rule: $\frac{dy}{dx} 3x^4 = 12x^3$

1(d)
$y = \pi$
Derivative of any constant value is 0: $\frac{dy}{dx} \pi = 0$

Sum Rule

Given Function $F(x) = f(x) + g(x)$
The derivative is $F'(x) = f'(x) + g'(x)$

I know in notation form it is rather complicated so I am going to explain it in relatively simple terms.

Imagine you have the following function
$f(x) = ax^n + bx^m$

To find the derivative, you must first split the function into two sections:

First Section	Second Section
ax^n	bx^m

Differentiate both separately to get

Derivative of first section	Derivative of second section
anx^{n-1}	bmx^{m-1}

Combine them back together to get your "overall" derivative

$f'(x) = anx^{n-1} + bmx^{m-1}$

If you still don't get it, take note of the following examples, often seeing how a real question work out clears most of your doubts.

Question 2

Differentiate the following with respect to x

(a) $y = 6x^7 + 2x^3$

(b) $y = 15x^2 + 4x^{-2} + \frac{1}{x}$

2(a)

Split the equation into two sections:

$\frac{dy}{dx} 6x^7 + 2x^3 = \frac{d}{dx} 6x^7 + \frac{d}{dx} 2x^3$

Differentiate the components one by one to get

$\frac{d}{dx} 6x^7 = 42x^6$	$\frac{d}{dx} 2x^3 = 6x^2$

Combine them back together to get the following

$\frac{dy}{dx} = 42x^6 + 6x^2$

2(b)

Convert all to index notation

$15x^2 + 4x^{-2} + x^{-1}$

Split equation into three sections

$\frac{dy}{dx} 15x^2 + 4x^{-2} + x^{-1} = \frac{d}{dx} 15x^2 + \frac{d}{dx} 4x^{-2} + \frac{d}{dx} x^{-1}$

Differentiate the components one by one to get

$\frac{d}{dx} 15x^2 = 30x$	$\frac{d}{dx} 4x^{-2} = -8x^{-3}$	$\frac{d}{dx} x^{-1} = -1(x)^{-2} = -x^{-2}$

Combine them to get

$\frac{dy}{dx} = 30x - 8x^{-3} - x^{-2}$

Difference Rule
Given Function $F(x) = f(x) - g(x)$
The derivative is $F'(x) = f'(x) - g'(x)$

Imagine the following function:

$$f(x) = ax^n - bx^m$$

To find the derivative split the function into 2 sections

First Section	Second Section
ax^n	$-bx^m$

Differentiate both section separately to get

anx^{n-1}	$-bmx^{m-1}$

Combine them back together to get your "overall" derivative

$f'(x) = anx^{n-1} - bmx^{m-1}$

Question 3:
Differentiate the following with respect to x
(a) $y = 5x^7 - 2x^3 - 7$
(b) $y = 3x^4 - 4x^2 + 5x^{-4}$

3(a)
Split equation into 3 sections to get
$\frac{dy}{dx} 5x^7 - 2x^3 - 7 = \frac{d}{dx} 5x^7 - \frac{d}{dx} 2x^3 - \frac{d}{dx} 7$

Differentiate the components one by one to get

$\frac{d}{dx} 5x^7 = 35x^6$	$\frac{d}{dx} - 2x^3 = -6x^2$	$\frac{d}{dx} 7 = 0$

Combine them back and you should get the following
$\frac{dy}{dx} = 35x^6 - 6x^2$

3(b)
Split equation into 3 sections to get
$\frac{dy}{dx}3x^4 - 4x^2 + 5x^{-4} = \frac{d}{dx}3x^4 - \frac{d}{dx}4x^2 + \frac{d}{dx}5x^{-4}$

Differentiate components one by one to get

$\frac{d}{dx}3x^4 = 12x^3$	$\frac{d}{dx} - 4x^2 = -8x$	$\frac{d}{dx}5x^{-4} = -20x^{-5}$

Combine them back to get the following:
$$\frac{dy}{dx} = 12x^3 - 8x - 20x^{-5}$$

Product Rule
Given Function $F(x) = f(x) \cdot g(x)$
The derivative is $F'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Imagine the following function
$f(x) = (ax^n)(bx^m)$
$f'(x) = anx^{n-1}(bx^m) + ax^n(bmx^{m-1})$

Question 4
Differentiate the following with respect to x
(a) $y = (x^2 + 15)(6x^4 + 9)$
(b) $y = (5x^9 + 6x - 5)(2x^2 + 6)$

4(a)
$y = (x^2 + 15)(6x^4 + 9)$
$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 15) \cdot (6x^4 + 9) + \frac{d}{dx}(6x^4 + 9) \cdot (x^2 + 15)$$

$$\frac{dy}{dx} = 2x(6x^4 + 9) + 24x^3(x^2 + 15)$$
$$= 12x^5 + 18x + 24x^5 + 360x^3$$
$$= 36x^5 + 360x^3 + 18x$$

4(b)
$y = (5x^9 + 6x - 5)(2x^2 + 6)$
$$\frac{dy}{dx} = \frac{d}{dx}(5x^9 + 6x - 5) \cdot (2x^2 + 6) + \frac{d}{dx}(2x^2 + 6) \cdot (5x^9 + 6x - 5)$$

$$\frac{dy}{dx} = (45x^8 + 6)(2x^2 + 6) + (4x)(5x^9 + 6x - 5)$$

$$= (90x^{10} + 12x^2 + 270x^8 + 36) + (20x^{10} + 24x^2 - 20x)$$
$$= 110x^{10} + 270x^8 + 36x^2 - 20x + 36$$

Quotient Rule

Given function $F(x) = \frac{f(x)}{g(x)}$

The derivative is $F'(x) = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{[g(x)]^2}$

Imagine you have the following function

$f(x) = \frac{ax^n}{bx^n}$

$$f'(x) = \frac{anx^{n-1}\,(bx^n) - bnx^{n-1}(ax^n)}{(bx^n)^2}$$

Question 5

Differentiate the following with respect to x

(a) $y = \frac{2x^5+3x^2-7}{(2x+6)}$

$$y = \frac{2x^5 + 3x^2 - 7}{(2x + 6)}$$

$$\frac{dy}{dx} = \frac{\left[\frac{d}{dx}\left(2x^5+3x^2-7\right) \cdot (2x+6) - \frac{d}{dx}(2x+6) \cdot \left(2x^5+3x^2-7\right)\right]}{(2x+6)^2}$$

$$\frac{dy}{dx} = \frac{\left[(10x^4+6x)(2x+6) - 2(2x^5+3x^2-7)\right]}{(2x+6)^2}$$

$$\frac{dy}{dx} = \frac{20x^5+12x^2+60x^4+36x-4x^5-6x^2+14}{(2x+6)^2}$$

$$\frac{dy}{dx} = \frac{16x^5+6x^2+60x^4+36x+14}{(2x+6)^2}$$

$$\frac{dy}{dx} = \frac{16x^5+60x^4+6x^2+36x+14}{\left(2(x+3)\right)^2}$$

$$\frac{dy}{dx} = \frac{16x^5+60x^4+6x^2+36x+14}{4(x+3)^2}$$

$$\frac{dy}{dx} = \frac{8x^5+30x^4+3x^2+18x+7}{2(x+3)^2}$$

Chain Rule
Given function
$F(x) = [f(x) + g(x)]^c$ {where c is a constant.}

The derivative is
$F'(x) = c[f(x) + g(x)]^{c-1} \cdot [f'(x) + g'(x)]$

Imagine you have the following function
$f(x) = (ax^n + bx^m)^c$
The derivative is
$f'(x) = c(ax^n + bx^m)^{c-1}\,[anx^{n-1} + bmx^{n-1}]$

Question 6
Differentiate the following with respect to $\boldsymbol{x}$
(a) $y = (2x^3 - 5x^2 + 2x)^7$

$y = (2x^3 - 5x^2 + 2x)^7$

$\frac{dy}{dx} = 7(2x^3 - 5x^2 + 2x)^{7-1} \cdot (6x^2 - 10x + 2)$

$\frac{dy}{dx} = 7(2x^3 - 5x^2 + 2x)^6\ \ (6x^2 - 10x + 2)$

Tips and Tricks Below
Question 7
Differentiate the following with respect to x

$$y = \frac{1}{(x+9)^5}$$

Question: Should I use chain rule or quotient rule for this question?
Answer: Chain rule, it is easier to use chain rule as demonstrated below.

Reason Below:
By Chain Rule (3 Steps)

$$y = \frac{1}{(x+9)^5} = (x+9)^{-5}$$

$$\frac{dy}{dx} = -5(x+9)^{-6}(1)$$

$$\frac{dy}{dx} = -5(x+9)^{-6}$$

By Quotient Rule (4 Steps)**:**

$$y = \frac{1}{(x+9)^5}$$

$$\frac{dy}{dx} = \frac{\left[0(x+9)^5 - 5(x+9)^{5-1}(1)\right]}{(x+9)^{5(2)}}$$

$$\frac{dy}{dx} = \frac{-5(x+9)^4}{(x+9)^{10}}$$

$$\frac{dy}{dx} = -5(x+9)^{4-10}$$

$$\frac{dy}{dx} = -5(x+9)^{-6}$$

Conclusion: If the numerator $=$ 1 and the denominator is in the form of $(\boldsymbol{ax^n} + \boldsymbol{bx^m} \ldots + \boldsymbol{c})$, you just have to convert the expression or equation into index notation and use chain rule to differentiate. (This only applies when the numerator = 1.)

Title	Additional Mathematics ['N' Levels] and Computing Mathematics Calculus – Basic Integration and Indefinite Integrals
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	7/4/2018

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The following are covered in this release:

Basic Overview of Integration

- Integration as the reverse of differentiation
- Notation guide
- Indefinite integral

Rules of integration

- Constant Multiple Rule
- Sum Rule
- Difference Rule

***Remember to print page 3 if you need something to refer to while doing your assignments. This material is not designed in the same way as the differentiation topic notes I created previously.

Integration as the reverse process of differentiation

As covered in earlier guide, we discussed how differentiation works, the reason why we study integration is the following reasons:

- There will be situations where finding the "anti-derivative" (AKA integral) is useful, if you are only given the gradient and ask to find out what is the equation of the function.
- Further study of integration will also be applied to finding the area under the graph of the function, using definite integral, which is useful for solving certain problems in various fields like Physics, Engineering, Chemistry and Information Technology.

Notation of Indefinite Integral

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

"Indefinite Integral of Function" Symbol

Arbitrary Constant

Explanation

Arbitrary Constant: The reason why we must have a "+C" to any indefinite integral is

- Reversing the integration process by differentiating the integral will result in many possible values of the constants that satisfy the integral, thus a "$+C$" is added, to represent "Constant"

Rules of Integration:

Integration of Constant Values

$$\int k\,dx = kx + C$$

Constant Multiple (Power Function Integration) Rule (Provided $n \neq -1$):

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int kx^n\;dx = k\int x^n = k\left(\frac{x^{n+1}}{n+1}\right) + C = \frac{kx^{n+1}}{n+1} + C$$

$\int (ax+b)^n\;dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$ (Provided $a \neq 0\ AND\ n \neq -1$)

Generalized Sum Rule

$$\int k \cdot f(x) + h \cdot g(x) = k\int f(x) + h\int g(x) + C$$

Generalized Difference Rule

$$\int k \cdot f(x) - h \cdot g(x) = k\int f(x) + h\int g(x) + C$$

Generalization of Constant Multiple, Sum and Difference Rule (Provided $n \neq -1$):

$$\int kx^n \pm gx^m = k\left(\frac{x^{n+1}}{n+1}\right) \pm g\left(\frac{x^{n+1}}{n+1}\right) + C = \frac{kx^{n+1}}{n+1} \pm \frac{gx^{n+1}}{n+1} + C$$

Indefinite Integral

Example 1:

Find the following integrals [Power Function Integration Rule]

(a)$\int x^6 \ dx$

(b)$\int \frac{1}{x^8} \ dx$

1(a) $\int x^6 \ dx = \frac{x^{6+1}}{6+1} = \frac{x^7}{7} + C$

1(b) $\int \frac{1}{x^8} \ dx = \int x^{-8} = \frac{x^{-8+1}}{-8+1} = \frac{x^{-7}}{-7} + C = -\frac{1}{7} x^{-7} + C$

Example 2:

Find the following integrals [Integration of Constant Values]

(a) $\int 15 \, dx$
(b) $\int 25 \, dx$

2(a) $\int 15 \, dx \ = 15x + C$
2(b) $\int 25 \, dx = 25x + C$

Example 3:

Find the following integrals [Constant-Multiple (Power Function Integration) Rule]

(a) $\int 2x^5 \, dx$
(b) $\int 16x^7 \, dx$

3(a) $\int 2x^5 \, dx$

$$\int 2x^5 \, dx = 2 \int x^5 \, dx + C = 2\left(\frac{x^{5+1}}{5+1}\right) + C = \frac{2x^{5+1}}{6} = \frac{2x^6}{6} + C = \frac{1}{3} x^6 + C$$

3(b) $\int 16x^7 \, dx$

$$\int 16x^7 \, dx = 16 \int x^7 \, dx = 16\left(\frac{x^{7+1}}{7+1}\right) + C = \frac{16x^8}{8} + C = 2x^8 + C$$

Example 4:

Find the following integrals [Generalized Sum Rule]

(a) $\int 2x^9 + 5x^3\,dx$

(b) $\int 9x^7 + 6x^{-8} + 12x + 7\,dx$

4(a) $\int 2x^9 + 5x^3\,dx$

$$= 2\int x^9 + 5\int x^3$$

$$= 2\left(\frac{x^{9+1}}{9+1}\right) + 5\left(\frac{x^{3+1}}{3+1}\right) + C$$

$$= 2\left(\frac{x^{10}}{10}\right) + 5\left(\frac{x^4}{4}\right) + C$$

$$= \frac{1}{5}x^{10} + \frac{5}{4}x^4 + C$$

4(b) $\int 9x^7 + 6x^{-8} + 12x + 7\,dx$

$$= 9\int x^7 + 6\int x^{-8} + 12\int x + \int 7$$

$$= 9\left(\frac{x^{7+1}}{7+1}\right) + \ 6\left[\frac{x^{(-8)+1}}{(-8)+1}\right] + 12\left(\frac{x^{1+1}}{1+1}\right) + 7x + C$$

$$= \frac{9}{8}x^8 + 6\left(\frac{x^{-7}}{-7}\right) + \frac{12x^2}{2} + 7x + C$$

$$= \frac{9}{8}x^8 - \frac{6}{7}x^{-7} + 6x^2 + 7x + C$$

Example 5

Find the following integrals [Generalized Difference Rule]

(a) $\int 2x^2 - 9x - \frac{2}{x^9} - 8 \ dx$

5(a)

$$\int 2x^2 - 9x - \frac{2}{x^9} - 8 \, dx$$

$$= 2\int x^2 - 9\int x - 2\int x^{-9} - \int 8$$

$$= 2\left(\frac{x^{2+1}}{2+1}\right) - 9\left(\frac{x^{1+1}}{1+1}\right) - 2\left(\frac{x^{-9+1}}{-9+1}\right) - 8(x) + C$$

$$= 2\left(\frac{x^3}{3}\right) - 9\left(\frac{x^2}{2}\right) - 2\left(\frac{x^{-8}}{-8}\right) - 8(x) + C$$

$$= 2\left(\frac{x^3}{3}\right) - 9\left(\frac{x^2}{2}\right) + 2\left(\frac{x^{-8}}{8}\right) - 8x + c$$

$$= \tfrac{2}{3}x^3 - \tfrac{9}{2}x^2 + \tfrac{2}{8}x^{-8} - 8x + C$$

$$\frac{2}{3}x^3 - \frac{9}{2}x^2 + \frac{1}{4}x^{-8} - 8x + C$$

Example 6

Find the following integrals [Generalization of all Rules Mentioned Combined]

(a) $\int 3x^5 - 6x^4 + 11x - 16 + 9x^{-7}\ \ dx$

$$3\int x^5 - 6\int x^4 + 11\int x - \int 16 + 9\int x^{-7}$$

$$3\left(\frac{x^{5+1}}{5+1}\right) - 6\left(\frac{x^{4+1}}{4+1}\right) + 11\left(\frac{x^{1+1}}{1+1}\right) - 16(x) + 9\left(\frac{x^{-7+1}}{-7+1}\right) + C =$$

$$3\left(\frac{x^6}{6}\right) - 6\left(\frac{x^5}{5}\right) + 11\left(\frac{x^2}{2}\right) - 16x + 9\left(\frac{x^{-7+1}}{-7+1}\right) + C =$$

$$\frac{3}{6}x^6 - \frac{6}{5}x^5 + \frac{11}{2}x^2 - 16x + \frac{9x^{-6}}{(-6)} + C =$$

$$\frac{1}{2}x^6 - \frac{6}{5}x^5 + \frac{11}{2}x^2 - 16x - \frac{9}{6}x^{-6} + C =$$

$$\frac{1}{2}x^6 - \frac{6}{5}x^5 + \frac{11}{2}x^2 - 16x - \frac{3}{2}x^{-6} + C$$

Title	Additional Mathematics ['N' Levels] and Computing Mathematics – Finding the Definite Integral of a Function
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Disclaimer: This file does not cover logarithmic functions, exponential function and trigonometric function.

You need the following prerequisite knowledge before proceeding

- Able to find indefinite integral of a function
- Understand the notation of indefinite integral

Notation used in finding definite integral	
$$\int_h^g f'(x) = \lvert f(g) - f(h) \rvert$$	
g and h	Refers to the boundary between the two points on the x-axis you are finding the area under graph for
$\lvert n \rvert$	This is sometimes referred to as the absolute value of n, any negative value that is between the two vertical strokes becomes positive while positive value within the vertical strokes remains positive.

Example 1
Find the following definite integral

$$\int_0^2 x^2 + 1\, dx =$$

Indefinite integral of the mentioned function is:	$\int_0^2 x^2 + 1\, dx = \frac{x^3}{3} + x$
Since we are finding the definite integral, we rewrite the integral in the following notation	$\left[\frac{x^3}{3} + x\right] \begin{matrix} 2 \\ 0 \end{matrix}$
Substitute 0 and 2 into the value x.	$\left\|\frac{2^3}{3} + 2 - \left(\frac{0^3}{3} + 0\right)\right\| =$
Final Answer	$\frac{14}{3} units$

Example 2

Find the following definite integral

$$\int_0^1 4 + 3x^2 \, dx$$

The indefinite integral of the function is	$\int_0^1 4 + 3x^2 = 4x + \frac{3x^3}{3}$
Rewrite using the following notation	$\left[4x + \frac{3x^3}{3}\right]\begin{matrix}1\\0\end{matrix}$
Substitute 0 and 1 into the value of x	$\left\vert 4(1) + \frac{3(1)^3}{3} - \left(4(0) + \frac{3(0)^3}{3}\right)\right\vert$
Final Answer	$5\ units$

Example 3
Find the following definite integral
$$\int_{7}^{0} 6x^7 + 2x - 6x^2 + 5$$

The indefinite integral of the function is	$\int_{7}^{0} 6x^7 + 2x - 6x^2 + 5 = \frac{6x^8}{8} + \frac{2x^2}{2} - 6\left(\frac{x^3}{3}\right) + 5x$
Rewrite using the following notation	$\left[\frac{6x^8}{8} + \frac{2x^2}{2} - 6\left(\frac{x^3}{3}\right) + 5x\right]\begin{matrix}0\\7\end{matrix}$
Substitute 0 and 7 into the value of x	$\left\|\frac{6(0)^8}{8} + \frac{2(0)^2}{2} - 6\left(\frac{0^3}{3}\right) + 5(0) - \left[\frac{6(7)^8}{8} + \frac{2(7)^2}{2} - \frac{6(7^3)}{3} + 5(7)\right]\right\|$
Final Answer	$= 4322998.75\ units$

Example 4

Find the following definite integral

$$\int_{10}^{2} \sqrt{x^3} + 2x - 9$$

Indefinite integral of the function is	$\frac{3}{4}x^{\frac{4}{3}} + x^2 - 9x$
Rewrite using the notation as shown	$\left[\frac{3}{4}x^{\frac{4}{3}} + x^2 - 9x\right]\begin{matrix} 2 \\ 10 \end{matrix}$
Substitute 2 and 10 to find the definite integral	$\left\|\frac{3}{4}(2)^{\frac{4}{3}} + 2^2 - 9(2) - \left[\frac{3}{4}(10)^{\frac{4}{3}} + 10^2 - 9(10)\right]\right\|$
Answer is	$38.268\ units$